Introduction to Themes (2)

Natural language is recognized as a cumbersome and completely inadequate means of expressing most mathematical argument and some significant logical argument. The information expressed in tabular form, in rows and columns, is superior to continuous prose as a means of expressing information in many cases, allowing comparisons to be made easily..

Philosophers (including logicians) make, of course, extensive use of symbolic notation in many philosophical as well as purely logical contexts. Tabular display is used in truth tables, the rows showing possible assignments of truth values to the arguments of the truth-functions or truth-functional operators. However, most philosophical argument is in continuous prose. I think that a symbolic notation has great utility in replacing philosophical prose in many contexts. This notation can be used to express established concepts and linkages but also new linkages and a new approach to 'philosophical taxonomy,' and in fact to the wider taxonomy of knowledge. I introduce these by giving a few examples of established mathematical, logical and philosophical terms and show the the possibility of generalizing them and extending them, using symbolic notation. In each case, I give further examples which are purposely chosen from fields seemingly remote from the starting point, and aim to show that the same concepts and symbolic notation can be applied to these fields.

To propose an extension of symbolic notation is only one of my aims here. The symbolic notation I propose has very little in common with Frege's 'Begriffsschrift.' It is incomparably less rigorous but has a far wider sphere of application (the examination and generalization of 'sphere of application' happens to be one of my aims.) I do share Frege's ambition, expressed in the Preface to the Begriffsschrift, 'if it is one of the tasks of philosophy to break the domination of the word over the human spirit by laying bare the misconceptions that through the use of language often almost unavoidably arise concerning the relations between concepts and by freeing thought from that which onlythe means of expression of ordinary language, constituted as they are, saddle it, then my ideography, further developed for these purposes, can become a useful tool for the philosopher.' Frege's ideography was typologically very difficult to implement. I have taken care to use only symbols which are typographically ready to hand.

Symbolism may supplement as well as replace an argument in natural language, symbolic notation being interspersed with natural language. An analogue would be the code view in a Web creation program such as Dreamweaver, which appends the HTML code required to convert a passage in natural language to the form which makes up a web page.

The examples I choose as starting points are (1) **mathematical relations, other mathematical connectives and philosophical relation** generalized to give a theory of linkage and the notation of 'linkage schemata.' The resulting theory and the linkage schemata are applicable to a very extensive subject matter - ethical, aesthetic, epistemological and other philosophical subject matter, but also non-philosophical subject matter. (2) **the scope of the universal or existential quantifier,** generalized to give a generalized theory of scope and 'sphere of application:' These are useful, to give only a single example, in discussions in legal philosophy (the jurisdiction of a legal system is interpreted as its sphere of application.) (3) by examining a variety of activities - these include manipulating a mathematical equation, instituting reforms in a political system - I arrive at the importance of **themes.** These too are provided with a symbolic notation. (4) By considering the **completion of a mathematical proof** and the completeness of a truth table, then I consider the usually more contentious matter of the completion of a 'survey,' (which again is given a notation.) A generalized halmos symbol finds application in the theory of surveys, which have innumerable applications.

The particular examples from established mathematical, logical and philosophical procedure which I use as starting points for generalization and extension could, obviously, be replaced by others. The starting points I discuss are not generally the ones which I used to arrive at any of the concepts or notation I discuss here.

Obviously, the notation I propose is exactly that, proposed and not established notation, unlike, for example, the accepted symbolic notation for logical connectives. In some cases, I use 'established symbols' such as '⊇' in a generalized sense, the established sense being regarded as a special case. It is necessary to distinguish the different uses of those symbols which are used (1) in an 'established' sense, for example ⊇ used to indicate the material conditional and (2) in the notation I propose. Although it is often clear from the context, I indicate that a symbol is used in sense (1) by enclosing it in angle brackets, < >. These are **declaration brackets,** making clear the interpretation of what is enclosed. So, <   indicates an established usage, the material conditional. Declaration brackets are an example of what I term an **indicator.** I list and explain this and other indicators later.

The typographic availability of established symbols is an argument for using them in new ways, rather than using symbols which are not so easily available, or which are not yet used at all. Angle brackets are available at every keyboard, and it is certainly convenient to make use of them, and also to to give them more than one function. I make use of angle brackets as a 'declaration indicator,' but also for a particularly important purpose, to indicate **generalized linkage** - which I explain below - in linkage schemata. In every case, the different uses carry no risk of confusion.

(1) The generalization of mathematical relations and philosophical relation. *Linkage schemata.*

R is a mathematical relation on the set S and x, y   S, so that in established symbolism, xRy. In established symbolism, the negation of xRy is xR'y.

If x and y are items in an algebraic equation, then they may be connected in various ways, such as the ones symbolized by =,   ,   , .

Although terms such as 'relation' and 'connected' are customarily used, I employ, always, the terms 'linkage' and 'linked' and other terms are special cases. So, mathematical relation is a special caso of linkage. In symbolic notation, I use angle brackets, < >. The items which are linked are shown within contents brackets, [ ]. These form a **linkage schema.** The linkage schema is shown symbolically as [ ] < > [ ] The discussion here is confined to dyadic linkages.

If the simple mathematical example xRy is brought within the scope of Linkage Theory, and a linkage schema is used, then it is necessary to use declaration brackets. So:

[<x>] << R >> [<y>]. The outer linkage brackets have within them the inner declaration brackets, showing the dual use of angle brackets. This causes no difficulty or ambiguity.

I use the very convenient symbolic notation > < to indicate lack of linkage, so that [A] > < [B] is the symbolic way of indicating that [A] is not linked with [B]. So, instead of using the established mathematical notation R' in x R' y I use the schema [ ] > < [ ] Again, using declaration brackets,
[<x>] > <R > < [<y>]

Like mathematical relations, philosophical relation is also a special case of linkage, but now there are contentious aspects. Disagreement is possible. The Platonic view of philosophical relations is very different from the nominalistic view, reflecting different views of properties. **Citation brackets, (+...)** can be given within the linkage brackets. Since again curved brackets have more than one use in notation on account of their general usefulness, I distinguish citation brackets by the inclusion of the + sign. Citation brackets may give simply a source or authority, such as (+Plato) or may give more detailed information. Often, this information will be too lengthy to include the citation brackets themselves, so a reference - for example a numbered reference - may be made to a source of information below the brackets. For example, numbers 1, 2, 3... May be references to philosophical journals.

A non-philosophical example:
[Smoking] < (+ 1, 2, 3) > [increased risk of lung disease]
Where 1, 2, 3 are references to scientific journals which examine the linkage between smoking and the increased risk of lung disease which is claimed.

Citation brackets are a particular case of **expansion brackets,** used where it is necessary to provide further information, either within the contents brackets or the linkage brackets. They may be used to note a qualification to what is stated in the brackets, to reduce ambiguity, and for many other purposes. Again, expansion brackets take the form (+...)

I speak of the contents of linkage brackets < > as well as the contents of contents brackets [ ]. The contents of the contents brackets are ontologically general and they may include processes, events and activities as well as objects. Contents brackets may also contain linkages, in discussions of the linkage between certain linkages. The contents of the linkage brackets are general, but with restriction. Objects and entities may not be placed in linkage brackets.

The use of linkage schemata expresses many linkages with particular force:
[a priori knowledge] < certainty (+...) [knowledge based on immediate sensory experience of a colour] The linkage schema need not be written on a single line. Often, as here, this is impractical. Here, the expansion brackets can point to a discussion (outside the linkage schema) of the certainty possessed by knowledge which is prior to or independent of experience, and knowledge which is based on sensory experience.

Given the few restrictions on the possible content of linkage and contents brackets, actual content is the result of further **restriction**.

The contents of the contents brackets may be restricted to the physical objects of the world and mental processes, in a ‘common-sense’ world view. Regarding this common-sense view as itself a metaphysical view, there are other metaphysical views which would give widely differing contents: the forms of Plato, the possible non-existents of Meinong, the monads of Leibniz, for example.

‘Restriction,’ to anticipate the discussion later, is a **theme** and I use the curly brackets which indicate a theme, {restriction}. {restriction} is symbolized as ‘==’ and ‘is applied to’ is symbolized as ‘:-’ so that if {restriction} is applied to the contents of contents brackets [A] in the linkage schema [A] < > [B] then this can be symbolized as == :- [A]

To return to the example with which this section began, == :- <<R>> gives such particular mathematical relations as < = > and < ≤ >.

{restriction} of the contents brackets leads to {restriction} of the linkage brackets. {restriction} of the linkage brackets leads to restriction of one, either or both of the contents brackets. For example, if the contents brackets contain macroscopic objects such as tables, then ‘gravitational attraction’ may be placed in the linkage brackets but a linkage by nuclear forces is excluded.

I use the symbol ⊃ to symbolize ‘leads to,’ as well as the sense ‘if...then’ which is close to the restricted logical use of the symbol. As already, mentioned, if ⊃ is used in this restricted sense, then it is declared and enclosed within declaration brackets.

I give now some non-technical examples which show that linkage schemata amongst their other advantages have the advantage of presenting a dilemma, a difference of opinion or what is claimed to be erroneous opinion, in a very clear form.

[Celestial objects, eg some star constellations] < (+astrological theory) [some events in human life] Whilst to the sceptic, [these celestial obects] > < [these events in human life] Again, the linkage schema need not be written on a single line.

[Artistic worth] > < [commercial success and overwhelming popularity] Again and again, claims are made to the contrary: so, being first in a list of best-sellers amounts to a claim about worth.

(2) The scope of the universal or existential quantifier. ***Sphere of application.***

The scope of the universal quantifier in (x) Fx ⊇ Gx is ‘Fx,’ the nearest complete expression to the right. The scope in longer logical statements can be shown by horizontal underlining. Underlining used to indicate scope may in some cases have the disadvantage that short sections of underlining in a Web document may be confused with the underlining which indicates a link. In the case of logical derivations, to the numbered formulas there can be added a vertical line to show scope, the ‘scope line.’ There may be a primary scope line and other scope lines. These examples of scope in logic are special cases of what I call ‘sphere of application’ and I see the need for a generalized theory of sphere of application. I use a different method of indicating logical scope and sphere of application. I use two asterisks, *...* which may be horizontal or vertical. So, if it is intended to show the scope of the universal quantifier in the example above, then I write (x) *Fx* ⊇ Gx Asterisks placed on an imaginary vertical line are typographically easier to implement, in general, than an actual vertical scope line.

Asterisks are a useful way of indicating ‘sphere of application,’ which can be regarded as a generalization of scope. In a passage, qualifications or a restriction may be announced, and their sphere of application can be shown by enclosing the qualified or restricted material within asterisks. In a substantial paragraph of such material, there will be an asterisk at the beginning and end of the paragraph.

Themes

The example of resolving power in optics. The viewer using a lens of insufficient resolving power will be unable to distinguish two points. They will seem to be one. If the lens is replaced with one of greater resolving power, then the points are separated. This exemplifies the theme {separation}. In the realm of pure ideas, an idea which seems to be one with a view of insufficient ‘resolving power’ can be separated into different components with a view which is more acute, has greater insight, with sufficient ‘resolving power.’ Both the empirical and the non-empirical example exemplify the theme {separation}.

Taking the simple equation a + b = c as a starting point, to arrive at the equation a = c - b then movement of b from the left side of the equation to the right side is necessary, but the movement is not fundamentally spatial movement, for example movement on the blackboard or computer screen used to display the modification of the equation. Nevertheless, to concentrate upon the more general level of modification rather than movement, both modification of an equation and modification of spatial position both exemplify ‘modification.’ I recognize the underlying similarity by including both in the same theme, {modification}, which has as one of its many sub-themes {spatial movement}. I indicate the sub-theme by writing {/movement}.

Reforms of an institution or political system also exemplify {modification}. The evaluated attitude to these reforms are strikingly different. Conservative thinkers may stress the risks of {modification}, claiming that there are more ways to ruin an institution or political system than to improve it. Others may stress the disadvantages of failing to implement {modification}, the continuance of lethargic or obviously unjust systems.

A mathematical ordered set is a sequence of elements distinguished by (1) the identity and (2) the order of elements. If a b then is not identical with

Compare this with the very distant sphere of applied ethics. Someone who happens to be convinced that so-called ‘speciecism’ is ethically mistaken may claim that the life of a dog and the life of a human are equally valuable, worthy of the same consideration - the unordered view. Others will come to the conclusion that ordering is necessary, and that the life of the human has to be placed before that of the dog. This is to implement the theme {ordering}, which has within its sphere of application matters as varied as numbers and lives.

Similarly, addition of biomass and addition of numbers belong to the same sub-theme, {/addition}, of the theme {modification}. {/Addition} and the earlier example of {/movement} are here both treated as sub-levels, but they are sub-levels only in the sense that they both have subordination to the higher level of {modification}. No attempt is made to assign themto a rigorous scheme and in fact this would hardly be possible. Ordering, is then, incomplete. In some instances of hierarchical organization, with restrictions placed upon the possible levels of organization, then ordering may be strict into level, /sub-level and //sub-sub-level. This is the case with computer files organized into directiories. If a complete ordering is given or attempted, then this is declared.

I concentrate now upon notation, including symbolic notation. In each case, the symbol indicates an activity, for example, 'to modify' but the name is a noun.

I use ‘ for {modification}read as 'to modify.' (The use of capital delta is suggested by its use in science for 'change of,' as in 'change of enthalpy.)

// for {separation}This symbol will not be confused with the symbol to indicate a sub-theme of a sub-theme, as in the example {modification} has {/expansion} and {/expansion} has {//diversification} since in this case ‘//’ appears immediately after ‘{’

== for {restriction} In symbolic notation, 'to restrict.'==(f) is 'free {restriction}', the activity of a free agent.
==(b) is 'bound {restriction}.

The {reversal} of {restriction} is {expansion}, shown as =/=, the crossing-out of {restriction}.

These, and other thematic symbols, may be used together with established connectives of symbolic logic, including the symbols of modal logic:

◊for 'possibility.'

Conjunction: 'and.'

Disjunction: 'or.'

~ Negation. ~~ Cancellation of negation.

⊂ The conditional: 'if...then.'

However, these symbols are used in the most general sense, not with their restricted logical sense. So ‘◊/ denotes contingent possibility as well as logical possibility. If the restricted logical sense is used, then the symbols are enclosed in declaration brackets <> as in <◊>.. The contents of the brackets are 'declared' as being of a restricted mathematical, logical or other sense.

⊃ is used in a non-restricted sense meaning 'to give' when it follows:- and the 'if...then' sense of the symbol is preserved: for example, if operation P is applied to entity X or theme Y then Z is the result. So, {restriction} when applied to the linkage symbol ⊇ gives the logical symbol < ⊇ > the brackets indicating that the symbol is not used in a generalized sense but its restricted logical sense. Symbolically, == :- (⊇) ⊇ < ⊇ > .Here, the brackets ( ) are used to indicate that '⊇ is the object of application. They can be omitted when the meaning is clear.

Examples of the use of thematic symbols and connectives

P 'to modify P,' {modification} of P. The thematic symbol is written before the variable, except in the case of //, written between the variables to be separated.

/ == is 'modification by restriction.'

/ =/= is 'modification by expansion.'

P // Q 'to separate P and Q.'

P // Q // R 'to separate P, Q and R'

◊ P // Q 'possible to separate P and Q ' or 'P and Q are separable.'

~ ◊P 'not possible to modify P,' or 'P is not modifiable.'

Q 'to order P is to order Q.' If P is ordered then Q is ordered.

~ ◊ (P Q) 'it's not possible to modify P and also to modify Q.'

◊P // Q R // S 'if it's possible to separate P and Q then it's possible to separate R and S.'

(4) Completion of a mathematical proof, truth tables and surveys

The completion of a mathematical proof is commonly symbolized by the solid or, sometimes, open square which mathematicians call the 'halmos' symbol, ('QED' or 'quad erat demonstrandum' serves the same function). I use here '☐' for this purpose. The symbol is a sign that the proof is complete, given certain qualifications to do with premises and axioms which I do not pursue here. The completeness of a truth table is established by the fact that all the rows of the table collectively give all the possible assignments of truth-values to these arguments. So, in the term I make use of, the **survey** is complete. A non-philosophical example.

A survey of the states of a digital, but not an analogue device, is simple: device is in either in the state ‘on’ or ‘off.’ ‘On’ and ‘off’ are **survey-items** in the survey and I show the survey-items, separated by commas, within a pair of curved brackets. So, the survey for the digital device A showing the possible states is:
((on, off □)). The Halmos symbol □ shows that the survey is complete.

In contentious surveys, surveys in which there are differences of opinion as regards the survey-items to be included and whether or not the survey is complete, then explanation and amplification, comment in general, will often be provided. This will be by means of amplification brackets, (+...) The bracketing notation for a survey does not allow space for a full presentation of the survey items, very often, but a short listing of the survey items will still be informative. In the example which follows, concerning the death penalty, the survey might begin,
((deterrence, reformation, retribution...))

A well-constructed survey - which may be an erroneous survey - often amounts to an attempt to gather together all, or as many as possible, of the considerations which are relevant to an issue. If the issue is the ethics of the death penalty, then the survey should be comprehensive, including, for example, the comparative costs of imprisonment and of proceeding to an execution. Often, views are based on one or more faulty interpretations of a survey-item, such as assuming that it is cheaper to execute than to imprison for life. (In the USA, this is not the case.) This survey may also include matters which are not ‘information,’ such as revulsion at the execution process, revulsion at the act of murder.

I use ‘ □ - ‘within survey brackets to indicate surveys which are widely, and unreflectingly, regarded as complete but which omit one or more survey-items which, it can be argued, are very significant. Because the completeness is ‘so-called’ completeness then the halmos symbol has ‘ ‘. These ‘missing’ survey items are given after the minus sign. These arguably defective surveys include many examples which are of immense importance in the contemporary world. Examples:

(1) ((ethical responsibility towards humans □ - ethical responsibility towards sentient beings in general. (+ Peter Singer))). The citation brackets give the name of just one philosopher who argues that ethical responsibility is owed to sentient beings in general.

(2) ((Survey items evaluated in terms of recycling □ - not evaluated in artistic/intellectual terms)) The very prevalent tendency to approve a book if printeed on recycled, unbleached paper. Whether the book's contents are valuable or trivial or even trashy is immaterial.

(3) ((Beliefs evaluated according to whether or not they lead to violence, acts of terroris □ - not evaluated according to whether or not they based on evidence/rational argument)) The tendency to concentrate only on social benefits or disadvantages and to ignore questions of validation.

The system of biological taxonomy which includes as levels kingdom, order, family, genus and species illustrates **diversification.** Diversification can be **global** or local.The system of biological taxonomy illustrates global diversification. Global and local diversification differ in their starting point. Global diversification is the equivalent of a map of an area, either large or small. Local diversification takes present (usually non-spatial) position as a starting point. Both global and local diversification may not be complete. Complete diversification is illustrated by the diversification schema for the states of a digital device, on or off. The discussion of concepts and their associated notation in this document is local rather than global. There are local starting points, such as mathematical relation.

The possibility of local diversification arises at innumerable points in an argument. Very often, this may take the form of adding to a statement the **diversification operator,** given as OR and symbolized as and the possibility of diversification is shown by means of diversification brackets, in which the appending is shown by means of a '+' sign: (+...).

The appending of diversified material (X below) will often cancel the faulty completeness of a faulty survey (again, X below.)

((...'□' - X)) and (+ X)

Very often, the **ordering** of survey-items will be attempted. There may be a claim to strict ordering: the ordering is claimed to be complete, as shown by the inclusion of the symbol □ after the survey-items, as in this example. The claim, which would be very widely accepted, far more so than any alternative, that in judging the claim upon our ethical commitment, in such a matter as saving life, we owe a greater commitment to a human person than to an ape and a greater commitment to an ape than to a mouse: ((human > ape > mouse (+ claims upon our ethical commitment, as, eg, in saving life) □